Exercise 57

If p is a polynomial, show that $\lim_{x\to a} p(x) = p(a)$.

Solution

Suppose that p is an nth-degree polynomial in x.

$$p(x) = c_0 + c_1x + c_2x^2 + \cdots + c_nx^n$$

Take the limit of $p(x)$ as $x \to a$.

$$\lim_{x\to a} p(x) = \lim_{x\to a}(c_0 + c_1x + c_2x^2 + \cdots + c_nx^n)$$

The limit of a sum is the sum of the limits.

$$\lim_{x\to a} p(x) = \lim_{x\to a} c_0 + \lim_{x\to a} c_1x + \lim_{x\to a} c_2x^2 + \cdots + \lim_{x\to a} c_nx^n$$

The limit of a constant times a function is the constant times the limit of the function.

$$\lim_{x\to a} p(x) = c_0 \lim_{x\to a} 1 + c_1 \lim_{x\to a} x + c_2 \lim_{x\to a} x^2 + \cdots + c_n \lim_{x\to a} x^n$$

The limit of a product is the product of the limits.

$$\begin{aligned}\lim_{x\to a} p(x) &= c_0 \lim_{x\to a} 1 + c_1 \lim_{x\to a} x + c_2 \left(\lim_{x\to a} x\right)^2 + \cdots + c_n \left(\lim_{x\to a} x\right)^n \\ &= c_0(1) + c_1(a) + c_2(a)^2 + \cdots + c_n(a)^n \\ &= c_0 + c_1a + c_2a^2 + \cdots + c_na^n \\ &= p(a)\end{aligned}$$